

Curvature

1. Consider a path in vector form $r(t) = \langle x(t), y(t), z(t) \rangle$:

(a) Write the equation for unit tangent vector $T(t)$

(b) Write the equation for curvature $\kappa(s)$

(c) Write the equation for curvature that uses cross product

2. Compute $\kappa(t)$ of the following:

(a) $r(t) = \langle 1, e^t, t \rangle$

(b) $r(t) = \langle 4 \cos(t), t, 4 \sin(t) \rangle$

(c) $y = t^4$, at $t = 2$

3. Normal Vectors

(a) In terms of $T(t)$, write the formula for unit normal vector.

(b) Find the normal vectors to $r(t) = \langle t, \cos(t) \rangle$ at $t = \frac{\pi}{4}$ and $t = \frac{3\pi}{4}$.

Limits of multivariable functions

1. Compute the following limits (they all exist):

$$\lim_{(x,y) \rightarrow (1,2)} (x^2 + y)$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x}{y}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$$

2. Compute the following partial derivatives

(a) $\frac{\delta}{\delta x} \frac{xy}{y^2+1}$

(b) $\frac{\delta}{\delta x} \frac{y^2}{y^2+1}$

(c) $\frac{\delta}{\delta x} \frac{\delta}{\delta y} \frac{x}{y}$

(d) $\frac{\delta}{\delta x} x^x$

3. Linearization

(a) Write the linear function for the linearization, $L(x, y)$, of $f(x, y)$ at (a, b)

(b) Let $f(x, y) = x^2y^3$. Find the linearization of f at $(a, b) = (2, 1)$

(c) Use (b) to estimate $f(2.1, 1.1)$.

4. Write the equation of the tangent plane to $z = f(x, y)$ at (a, b) .

5. Find the equation of the tangent plane at the given point.

(a) $z = x^2y + xy^3$, $(2, 1, 6)$

(b) $f(x, y) = e^x \ln(y)$, $(0, 1)$
